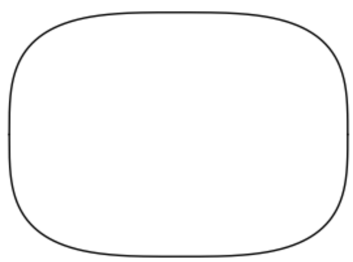
THE GOLDEN SUPERCIRCLE

In the familiar normed space ℓ_p, the distance from (x, y) to the origin is defined as $(|x|^p + |y|^p)^{1/p}$. Taking all such points with distance 1, we get something called a supercircle. These shapes, between squares and circles, have found many uses (see [2]) in design of buildings, fonts, roads, and app icons.

Each supercircle has a circumference (total arc length), which we can divide by its diameter to get π_p, the version of π for ℓ_p. We know some things about π_p (see [1] and the references therein) – it decreases from $\pi_1 = 4$ to its minimum $\pi_2 \approx 3.14159$ (the usual π), and then increases to $\pi_\infty = 4$. Since $\pi_x - x$ changes sign in $(2, \infty)$, by the intermediate value theorem there is some golden value $G \in (2, \infty)$ with $\pi_G = G$. By monotonicity this value is unique; we find

$$\pi_G = 3.3052415857\ldots.$$

This golden supercircle constant appears to be new, i.e. not expressible from other constants. The associated golden supercircle is pictured below.



References

[1] Keller, J. B. and Vakil, R. (2009). π_p, the Value of π in ℓ_p. *Amer. Math. Monthly.* **116** (10), 931–935.

[2] Supercircle. (2022). `https://en.wikipedia.org/wiki/Superellipse#History`.