Math 150 Spring 2012 Final Exam

Please read the following directions.

PLEASE DO NOT BEGIN THE EXAM UNTIL INSTRUCTED TO DO SO.

Books, notes, calculators, and other aids are not permitted on this exam, apart from a single note sheet. Please write legibly, with plenty of white space. If you need scratch paper or extra space, you may use the back of the exam. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade.

Problem	Your Grade	Max Grade
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10

Problem	Your Grade	Max Grade
11		10
12		10
13		10
14		10
15		10
16		10
17		10
18		10
19		10
20		10
TOTAL		200

(1) Find $\lim\limits_{x\to\infty} \dfrac{\sqrt{2x+\sqrt{5x}}}{\sqrt{3x+\sqrt{7x}}}$.

(2) Find the equation of the tangent line at $(1,9)$ to the curve $\sqrt{x}+\sqrt{y}=4$.

Problems (3) and (4) refer to $f(x) = \begin{cases} x + 1 & x < 2 \\ a & x = 2. \\ a^2 - ax & x > 2 \end{cases}$

(3) Determine for which values of a, if any, $\lim_{x \to 2} f(x)$ exists.

(4) Determine for which values of a, if any, $f(x)$ is continuous at $x = 2$.

(5) For $f(x) = x^{\sin x}$, find $f'(x)$.

(6) Find all a in $[0, 2\pi]$ where the tangents to $\frac{1}{2}\cos 2x$ and $\sin x$ (at a) have the same slope.

(7) How far does a ball drop in 3 seconds? Assume no air resistance, and that $g = 10$ m/s.

(8) Use the mean value theorem to prove that $x \geq \sin x$ for all $x \in [0, 6]$.

(9) You inflate a spherical balloon at the rate of $\frac{\pi}{25}$ liters/second. How fast is the radius growing when it is 1 decimeter? (a liter is a cubic decimeter)

(10) Find the point on the curve $x = 2y^2$ closest to $(\frac{1}{4}, 64)$.

(11) Find all local minima of $f(x) = x(x-1)^7$.

(12) Find all inflection points of $f(x) = x(x-1)^7$.

(13) Use differentials to estimate $\sqrt{101}$.

(14) Find $\lim\limits_{x\to\infty} \frac{2^{-x}}{x^{-1}}$.

(15) Find $\int \frac{1}{x} + \frac{1}{\sqrt{x}}\, dx$.

(16) Find $\int \frac{2\ln x}{x}\, dx$.

(17) Find $\int 2^x \sin 2^x \ dx$.

(18) Estimate $\int_0^{\pi/3} \sin 3x \ dx$ using a trapezoidal approximation with four subintervals.

(19) Calculate $\int_0^{\pi/3} \sin 3x \ dx$ exactly, using the fundamental theorem of calculus.

(20) Use Riemann sums and limits to find $\int_1^3 x^2 - 2x \ dx$. Do not use the FTC.