Math 254 Fall 2011 Exam 2a Solutions

1. Carefully state the definition of "linear function". Give two examples.

   A linear function is a function of one or more variables that consists entirely of multiplying its input variables by scalars, and adding the results. Many examples are possible, of course, such as $f(x) = 7x, g(x, y) = 7x, h(x, y) = 0$.

2. Solve the following system, using back-substitution. Show your work.

$$\begin{array}{rcrcrcrcr} 2x_1 & + & 3x_2 & + & 7x_3 & + & x_4 & = & 11 \\ & - & 5x_2 & - & 5x_3 & + & 2x_4 & = & 4 \\ & & & & 3x_3 & + & x_4 & = & 7 \\ & & & & & & 3x_4 & = & 21 \end{array}$$

   The last equation gives $x_4 = 7$. Substituting into the third equation gives $3x_3 + 7 = 7$, which gives $x_3 = 0$. Substituting into the second equation gives $-5x_2 - 5(0) + 2(7) = 4$, which gives $x_2 = 2$. Finally, substituting into the first equation gives $2x_1 + 3(2) + 7(0) + 7 = 11$, which gives $x_1 = -1$. Hence $(x_1, x_2, x_3, x_4) = (-1, 2, 0, 7)$ is the sole solution.

3. Compare slope-intercept form and standard form for lines in the plane, giving one advantage of each as compared to the other. Write the same line twice, once in each form.

   Advantages of slope-intercept form include that the slope and intercept are useful in drawing a graph by hand, and in interpreting the line geometrically. The principal advantage of the standard form is that all lines may be represented, including vertical ones. Many examples are possible, such as $x + y = 1$ (standard) $y = -x + 1$ (slope-intercept).

4. Find the line of best fit for the following set of points: $\{(0, 0), (2, 1), (1, 2), (3, 5)\}$.

   We first calculate $n = 4, \sum x = 6, \sum y = 8, \sum xy = 19, \sum x^2 = 14$. Hence we need to solve the 2×2 system $\{4b + 6m = 8, 6b + 14m = 19\}$. One way to solve this is to multiply the first equation by 3, the second by -2, and add. The solution is $b = -1/10, m = 7/5$. Hence the line of best fit is $y = \frac{7}{5}x - \frac{1}{10}$ or $y = 1.4x - 0.1$.

5. Consider the system of equations $\{2x - 3y = 4, kx + 6y = 5\}$. For which values of k does this have exactly one solution (and what is it)? For which values of k does this have no solution? For which values of k does this have infinitely many solutions?

   Taking twice the first equation and adding to the second, we get $(4 + k)x = 13$. If $k = -4$, this has no solutions. Otherwise, $x = \frac{13}{4+k}$, which we substitute into $3y = 2x - 4$ to get $3y = \frac{26}{4+k} - 4 = \frac{10-4k}{4+k}$. Hence $(x, y) = (\frac{13}{4+k}, \frac{10-4k}{12+3k})$ is the unique solution for $k \neq -4$. The system never has infinitely many solutions.