Name:

Math 254 Fall 2011 Final Exam

Please read the following directions:
Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 120 minutes.

1. Carefully state the definition of "dimension". Give two subspaces of $\mathbb{R}^4$, a one-dimensional one and a three-dimensional one.

2. Carefully state the definition of "vector space". Give two examples, neither of which may be $\mathbb{R}^n$ for any n.

3. Carefully state the definition of "independent". Give two examples from $\mathbb{R}^1$.

Problems 4 and 5 concern $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$.

4. Find A^{-1}.

5. Use A^{-1} to solve $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

Problems 6,7,8 concern $A = \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 1 & 3 & 0 & 6 & 10 \\ 0 & 1 & 1 & 2 & 1 \\ 1 & 0 & -3 & -4 & -1 \\ 1 & 0 & -3 & 0 & 7 \end{bmatrix}$.

6. Find all solutions to $A\left[\begin{smallmatrix} a & b & c & d & e \end{smallmatrix}\right]^T = \left[\begin{smallmatrix} 3 & 4 & 1 & 1 & 1 \end{smallmatrix}\right]^T$.

7. Find the row canonical form for A.

Recall that Problems 6,7,8 concern $A = \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 1 & 3 & 0 & 6 & 10 \\ 0 & 1 & 1 & 2 & 1 \\ 1 & 0 & -3 & -4 & -1 \\ 1 & 0 & -3 & 0 & 7 \end{bmatrix}$.

8. Find a basis for the nullspace of A. What does this tell us about the eigenvalue $\lambda = 0$?

Problems 9,10,11,12 concern the map $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (x + y, 2x + 2y)$.

9. Prove that f is a linear map.

Recall that problems 9-12 concern the map $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (x + y, 2x + 2y)$.
10. Find the kernel and image of f, and give a basis of each.

11. Find the representations $[f]_E$ and $[f]_S$ in the standard basis E and the basis $S = \{\left(\begin{smallmatrix}1\\2\end{smallmatrix}\right), \left(\begin{smallmatrix}2\\3\end{smallmatrix}\right)\}$, respectively.

12. Find all eigenvalues of f, and give their algebraic and geometric multiplicities.

Problems 13 and 14 concern the polynomial vector space $V = P_2(t)$ and the subspaces $S = Span(t+1, t^2+1), T = Span(t^2+t+2, t^2-t)$.

13. Find a basis for $S + T$.

14. Find a basis for $S \cap T$.

Problems 15 and 16 concern the matrix $A = \left(\begin{smallmatrix} 1 & 2 \\ 2 & 3 \end{smallmatrix}\right)$ and the bilinear form $< u, v >_A = u^T Av$.

15. Let $u = \left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right)$, $v = \left(\begin{smallmatrix} 1 \\ k \end{smallmatrix}\right)$. Determine for which k, if any, we have $< u, v >_A = 0$.

16. Prove that A is not positive definite. Is $< u, v >_A$ an inner product?