## Math 254 Fall 2012 Exam 9

Please read the following directions:
Please print your name in the space provided, using large letters, as "First LAST". Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

Extra credit may be earned by handing in revised work in class on Friday 11/16; for details see the syllabus. You will find this exam on the instructor's webpage soon.

1. Carefully state the definition of "subspace". Give two examples, each within $\mathbb{R}^{3}$.
2. Let $A, B, C$ be linear transformations on finite-dimensional vector space $V$. Suppose that $A$ is similar to $B$, and that $B$ is similar to $C$. Prove that $A$ is similar to $C$.

For each $k \in \mathbb{R}$, we define a linear transformation $f_{k}: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$, given by $f_{k}((a, b))=$ $(2 a+k b, a+3 b)$. The remaining three problems concern these functions $f_{k}$.
3. Determine the nullity of $f_{k}$, for each possible value of $k$.
4. Determine the matrix representation $\left[f_{k}\right]_{E}$, for the standard basis $E=\{(1,0),(0,1)\}$.
5. Determine the matrix representation $\left[f_{k}\right]_{S}$, for the basis $S=\{(1,2),(2,3)\}$.

