

Name:

Math 254 Fall 2014 Exam 9

Please read the following directions:

Please print your name in the space provided, using large letters, as “First LAST”. Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

Extra credit may be earned by handing in revised work in class on Wednesday 11/12; for details see the syllabus. You will find this exam on the instructor’s webpage later today.

1. Carefully state the definition of “spanning”. Give two spanning sets for \mathbb{R}^2 .

2. Suppose that U is a vector space with (finite) basis B . Suppose that F, G are two linear transformations from U to U . Prove that if $[F]_B = [G]_B$ then $F = G$.

The remaining three problems concern the vector space $V = \left\{ \begin{pmatrix} a & b \\ b & d \end{pmatrix} : a, b, d \in \mathbb{R} \right\}$, its basis $E = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$, and $F : V \rightarrow V$ given by $F : \begin{pmatrix} a & b \\ b & d \end{pmatrix} \rightarrow \begin{pmatrix} d & a+d-b \\ a+d-b & a \end{pmatrix}$.

3. Prove that $F^2 = F \circ F$ is the identity linear transformation.

4. Calculate $[F]_E$.

5. Find the row canonical form of $[F]_E$, and use this to determine the rank and nullity of F .