MATH 521A: Abstract Algebra
Homework 7: Due Oct. 26

1. List all polynomials in $\mathbb{Z}_3[x]$ of degree at most 1. Determine which are units and which are zero divisors.

2. List all polynomials in $\mathbb{Z}_4[x]$ of degree at most 1. Determine which are units and which are zero divisors.

3. Let R be a commutative ring with identity. Define $f : R[x] \to R$ via $f : a_0 + a_1 x + \cdots + a_n x^n \mapsto a_0$. Prove that f is a (ring) homomorphism and find its kernel and image.

4. Let R be a commutative ring with identity. Let $a \in R$ be nilpotent. Prove that $1_R - ax$ is a unit in $R[x]$.

5. Working in $\mathbb{Z}_3[x]$, find $\gcd(a(x), b(x))$, for $a(x) = x^3+x^2+2x+2$, $b(x) = x^4+2x^2+x+1$.

6. Working in $\mathbb{Z}[x]$, find $\gcd(a(x), b(x))$, for $a(x) = 3x^2+2$, $b(x) = 4x^4+2x^3+6x^2+4x+5$.

7. Working in $\mathbb{Z}_7[x]$, find $\gcd(a(x), b(x))$, for $a(x) = 3x^2+2$, $b(x) = 4x^4+2x^3+6x^2+4x+5$.

8. Working in $\mathbb{Z}_7[x]$, let $a(x) = 3x^2 + 2$, $b(x) = 4x^4 + 2x^3 + 6x^2 + 4x + 5$. Find $u(x), v(x)$ such that $\gcd(a(x), b(x)) = a(x)u(x) + b(x)v(x)$.

9. Working in $\mathbb{Z}_{10}[x]$, find two degree-1 polynomials whose product is $x + 7$.