

MATH 521A: Abstract Algebra
Homework 9: Due Nov. 29

1. Calculate the multiplication table for $\mathbb{Z}_5[x]/(x^2 + 4x + 1)$.
2. Calculate the multiplication table for $\mathbb{Z}_5[x]/(x^2 + 3x + 1)$.
3. Calculate the multiplication table for $\mathbb{Z}_5[x]/(x^2)$.
4. Calculate the multiplication table for $\mathbb{Q}[x]/(x^2 + 2)$.
5. Calculate the multiplication table for $\mathbb{Q}[x]/(x^2 - 2)$.
6. Calculate the multiplication table for $\mathbb{Q}[x]/(x^2 - 1)$.
7. * For each of the rings in problems 1-6, calculate the (multiplicative) inverse of $[x - 1]$, or prove it does not exist.
8. Let $f(x), g(x), p(x) \in F[x]$, where all three polynomials are nonconstants. Suppose that $f(x)g(x) = p(x)$. Prove that $[f(x)]$ is a zero divisor in $F[x]/(p(x))$.
9. Let $f(x), p(x) \in F[x]$, where both polynomials are nonconstants. Set $g(x) = \gcd(f(x), p(x))$. Prove that $[f(x)]$ is a unit in $F[x]/(p(x))$, if and only if $g(x)$ is a constant polynomial.
10. Determine, with proof, which of the rings in problems 1-6 are integral domains, and which are fields.