Math 522 Exam 9: 3/21/13
Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Notes, calculators, and the textbook are all permitted. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. You will earn between 25 and 50 points on each problem. You have 30 minutes.

You may earn extra credit by submitting revised answers to *both* of the following problems, by the next class day (3/26/13). Please see the syllabus for more details.

Theorem 1. Let $m, n \in \mathbb{N}$. If $\gcd(m, n) = 1$ then $\phi(mn) = \phi(m)\phi(n)$.
Theorem 2. Let $p, k \in \mathbb{N}$. If p is prime, then $\phi(p^k) = p^k - p^{k-1}$.

1. Use the two theorems above to prove the following:
   Claim. For all $n \in \mathbb{N}$, $\phi(n) = n \prod_{p \mid n} \left(1 - \frac{1}{p}\right)$.

2. Compute $\phi(150), d(150)$, and $\sigma(150)$.