MATH 579: Combinatorics

Homework 4: Due Oct.2

1. Let $n \in \mathbb{N}_0$. Prove that $2^n = \displaystyle\sum_{i=0}^{n} \binom{n}{i}$.

2. Let $n \in \mathbb{N}_0$. Prove that $\dfrac{3^n + (-1)^n}{2} = \displaystyle\sum_{\substack{i=0 \\ i \text{ even}}}^{n} 2^i \binom{n}{i}$.

3. Let $n \in \mathbb{N}_0$. Prove that $\dfrac{6^n - (-4)^n}{2} = \displaystyle\sum_{\substack{i=1 \\ i \text{ odd}}}^{n} 5^i \binom{n}{i}$.

4. Let $n \in \mathbb{N}_0$. Prove that $n2^{n-1} = \displaystyle\sum_{i=0}^{n} i \binom{n}{i}$.

5. Let $n \in \mathbb{N}_0$. Prove that $\dfrac{1}{n+1} = \displaystyle\sum_{i=0}^{n} \frac{(-1)^i}{i+1} \binom{n}{i}$.

6. How many different acronyms does MISSISSIPPI have? (Note: it doesn't matter if the word appears in any dictionary)

7. Let $n \in \mathbb{N}_0$. Prove that $3^n = \displaystyle\sum_{i+j+k=n} \binom{n}{i,j,k}$.

8. Let $n \in \mathbb{N}_0$. Prove that $1 = \displaystyle\sum_{i+j+k=n} (-1)^i \binom{n}{i,j,k}$.

9. What is the largest coefficient in $(x_1 + x_2 + x_3 + x_4 + x_5)^{150}$?