MATH 579 Exam 5; 3/13/12
Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You have 40 minutes. If you wish, when handing in your exam you may attach your extra credit problem. For more details, see the syllabus.

Choose three problems only from these five.

1. (5-8 points) For all integers k, n with $1 \leq k \leq n$, prove that $\left\{ {n \atop k} \right\} = \left\{ {n-1 \atop k-1} \right\} + k \left\{ {n-1 \atop k} \right\}$,

$$\text{i.e. } S(n,k) = S(n-1, k-1) + kS(n-1, k).$$

2. (5-10 points) Prove that $S(n,3) = \frac{3^{n-1} - 2^n + 1}{2}$ for all $n \in \mathbb{N}$. Hint: induction on n.

3. (5-10 points) Use difference calculus to calculate $\sum_{k=1}^{19} k^3 - k$.

4. (5-10 points) We call a number "happy" if it yields remainder 2 upon division by 3. (Happy numbers include 2,5, and 8). How many compositions are there of 100 into 5 happy parts?

5. (5-12 points) How many compositions are there of 30 into 5 parts, one of which is happy and four of which are odd?