# MATH 579 Exam 9; 5/1/12
Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You have 40 minutes. If you wish, when handing in your exam you may attach your extra credit problem. For more details, see the syllabus.

**Choose three problems only from these five.**

1. (5-8 points) Use generating functions to solve the recurrence given by $a_0 = 3, a_n = 3a_{n-1} - 4$ ($n \geq 1$).

2. (5-10 points) Use generating functions to solve the recurrence given by $a_0 = 1, a_1 = 2, a_n = 2a_{n-1} - a_{n-2}$ ($n \geq 2$).

3. (5-10 points) $A(x) = \frac{x^4 + 5x - 2}{(1-2x)^5}$ is the generating function for a sequence $a_n$. Find a closed form for $a_n$ (for $n \geq 4$ is sufficient).

4. (5-10 points) Find a generating function $V(x)$ that can be used to count nonnegative integer solutions to $a + b + c = n$, where (1) $2 \leq a \leq 5$, (2) $b$ is a multiple of 5, (3) $c$ is odd. You should simplify $V(x)$, but:
   DO NOT ATTEMPT TO FIND A CLOSED FORM FOR THE SEQUENCE.

5. (5-12 points) Vadim's Amazing Magic Trick begins with a deck of $n$ cards. Someone from the audience is chosen to be the assistant. This assistant separates the deck into any number of nonempty piles. From each pile the assistant chooses one card to memorize. Also, the assistant places the Magic Vodka Bottle on top of one of the piles. Let $v_n$ denote the number of ways the assistant can do all this. $v_0 = 1, v_1 = 1, v_2 = 4$ (if two piles, two choices for MVB; if one pile, two choices for card to memorize). Find the generating function $V(x)$.
   DO NOT ATTEMPT TO FIND A CLOSED FORM FOR THE SEQUENCE.